Decision-making and Coordination System of Low-carbon Supply Chain Based on New Retail of Agricultural Products

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Abstract
Considering the impact of supplier carbon emission reduction levels, new retailer service efforts and traditional retailer promotion efforts on product demand, this paper proposes a new supply chain for a supplier, a new retailer and a traditional retailer, and introduces the new retail channel preference coefficient to construct a supply chain decision model under different contracts to analyze how each member can improve carbon emission reduction level, service effort level and promotion effort level through effective coordination mechanism. The research shows that the cost-sharing model enables the Pareto improvement of the supply chain to the greatest extent. The greater the marginal benefit of the supplier is, the worse the coordination effect of the cooperative emission reduction mode is. The degree of coordination between the cooperative service mode and the cooperative promotion mode depends on the impact of the level of service and promotion on channel demand. Channel retailers with a greater impact on channel demand for carbon reduction, will share higher carbon abatement costs. Supply chain profit is positively related to consumers’ preference for new retail channels, so opening up a "new retailing" model will help increase supply chain profit. With the development of e-commerce iteration, the retail industry has gradually changed to a data-driven "new retail" direction. At present, the scale of fresh e-commerce market represented by agricultural products is expanding constantly, and promoting the upgrading of agricultural supply chain is of great significance to accelerate the national economic and social development and reshape the business model of agricultural products industry. The "new retail" mode driven by big data and other network technologies provides new ideas and directions for the development of agricultural supply chain in the new era.

Key words: New retailing; Low Carbon Supply Chain; Contract Mechanism; Supply Chain Coordination; Agricultural Products.

1. Introduction
With the development of China’s economy and the increasing consumption level of residents, the demand for diversified retail mode is increasing. It has become a trend that improves the circulation efficiency of products through the deep integration of business mode and big data. Under this background, the ‘new retailing’ mode arises from the historic moment. The new retailing enterprise consists of online and offline dual channel, and inherits the characteristics of the O2O mode. ‘New retailing’ mode presents new requirements of multi-channel collaboration for supply chain management, and emphasizes the quality of logistics service, so the ‘new retailing’ mode fundamentally provides consumers with higher levels of service. At the same time, environmental pollution and resources shortage problems appeared gradually. In order to achieve the coordinated development between economy and environment, our country formulates a series of emissions reduction plans and promotes consumer environmental awareness. On September 3, 2016, before the opening of the G20 Summit, China deposited the ratification document of the Paris Agreement on Climate Change in China with the United Nations and Promised to make carbon dioxide emissions peak around 2030 [1]. It shows that enterprises still have huge room for improvement in energy conservation and emission reduction.

Compared with traditional retail, experiential consumption is the core of new retailing, the introduction to the new retail mode makes traditional retail faced intense competition. Traditional retailers guarantee their market share by offering discounts so that reducing channel conflict and enhancing its competitiveness. In the context of traditional retail and new retail competition, we look for the contract which can be better coordination of channel competition and maximize profits of the supply chain, which has become a new topic of theory research and practice of supply chain management.

2. Literature Review
The emission reduction of supply chain is currently a research hotspot, which is carried out in two directions: emission reduction strategy and low-carbon supply chain contract design. Rao et al. [2] investigated
East Asian enterprises and proved that enterprises implementing emission reduction strategies could increase product sales and market share. Govindan et al. [3] believed that low-carbon production made the supply chain sustainable, and corporate carbon emission reduction behavior was a manifestation of fulfilling social responsibility. Liu et al. [4] focused on the impact of competition and consumer environmental awareness on the supply chain, and believed that member competition had competition between manufacturers and between manufacturers and retailers. Luo et al. [5] considered the demand for price and carbon emission reduction under the carbon limit mechanism, and studied the price and carbon emission reduction decision of two manufacturers under the conditions of perfect competition and cooperative competition respectively. Zhou et al. [6] studied the decision-making optimization of low-carbon supply chain management and analyzed the impact of different contract mechanisms on the optimal decision-making and coordination of low-carbon supply chains. Giannoccaro et al. [7] designed a revenue-sharing contract to coordinate a three-tier supply chain that would increase the profitability of all members of the supply chain. Zissis et al. [8] considered a secondary supply chain consisting of one manufacturer and one retailer. In the case of retailers having private information, the volume discount contract could coordinate the supply chain in the case of information asymmetry.

Promotions and services are important factors influencing consumers’ purchasing behavior, and some scholars have also conducted researches on these two elements. Kumar et al. [9] constructed a two-channel supply chain consisting of one manufacturer and one retailer, considering the impact of retail channel service efforts on the dual-channel supply chain. Mukhopadhyay et al. [10] studied the information sharing method between the upstream and downstream of the supply chain in the case of asymmetric service information in the multi-channel supply chain, in which the manufacturer dominated. Krishnan et al. [11] demonstrated the negative impact of rapid response to supply chain implementation on the level of retailer’s promotional efforts, and demonstrated that minimum acceptance of contracts, pre-purchase contracts, or exclusive transactions and rapid response to supply chains would simultaneously eliminate the negative impact of supply chain’s rapid response on retailers’ promotional efforts. Petro [12] studied supply chain optimization issues that were determined by retailers to make promotional efforts and suppliers to determine product quality. Wang et al. [13] constructed a differential game model based on the assumption that the supplier’s low carbon reputation and retailer’s promotional efforts were combined to analyze the impact of several cooperative contracts on the supply chain.

The concept of ‘new retail’ has not been put forward for a long time, so most of the relevant researches focus on the definition of ‘new retail’ and development trends, but there are few theoretical studies based on mathematical models. Considering that ‘new retail’ is developed and improved based on the O2O model, and the theoretical exploration of the O2O model is relatively mature, it is possible to explore the ‘new retail’ supply chain by drawing on the research results in the O2O field. Researches in this area focus on empirical research with management indicators at the core and the use of mathematical models to study supply chain decision making. Dumrongsiri et al. [14] showed that retail staff response speed and employee working hours could affect the profit of dual-channel supply chain, and increasing these two indicators could improve the profit of supply chain. Yan et al. [15] analyzed the impact of retail employee service attitudes on supply chain profits. Cai et al. [16] designed a reasonable contract mechanism to study the impact of the contract on O2O supply chain profit.

In summary, most of the existing supply chain research literatures don’t combine product reduction, product service and product promotion in the context of ‘new retail’. Therefore, this paper considers a supply chain consisting of a reduction supplier, a new retailer that provides experiential consumer services and a traditional retailer with promotional behavior. Under decentralized decision making, members of the supply chain are economically independent individuals, and they make supply chain decisions with the goal of maximizing their respective profits. However, the independent decision making will reduce supply chain efficiency. To this end, each member of the supply chain should coordinate supply chain through effective contracts. By studying the models of cooperative emission reduction, cooperative promotion and cooperative service decision-making of suppliers, retailers and new retailers, this paper provides an effective cost-sharing contract to find the contractual mechanism for supply chain members to maximize Pareto improvement and optimize supply, and optimizes collaboration among members of the supply chain and addresses inefficiencies in the supply chain, which is of great importance to the development of low-carbon supply chains under the ‘new retail’ trend.

3. Problem Description and Assumptions
3.1. Problem Description

This paper considers a low-carbon supply chain consisting of a supplier, a new retailer and a traditional retailer, as shown in Figure 1. The reduction of carbon emission is decided by suppliers. New retailers provide experiential consumption to consumers, so the new retailer's make the decision of the new retailer’s service.
Traditional retailers are trying to promote their products in order to keep their market share, and decide the traditional retailer’s promotion.

![Figure 1. Schematic diagram](image)

### 3.2 Assumptions

Combining with the actual situation, we simplify the complex conditions and make following assumptions about the model:

**Assumption 1.** Supply chain members are risk-neutral and completely rational, and suppliers have the capacity to meet market demand.

**Assumption 2.** This study doesn’t consider the impact of price on demand, and the marginal revenue of suppliers, new retailers and traditional retailers are respectively constant values $w_i, I_n, I_t$.

**Assumption 3.** Referencing to Yu et al. [17], the cost of carbon reduction is an increasing and convex function with the emission reduction. In this paper, we consider that the cost of carbon emission reduction can be expressed as a quadratic function with the carbon emission reduction. In the same way, the cost function of new retailer’s service and traditional retailer’s promotion is expressed as follows.

$$C(e) = \frac{1}{2} k_1 e^2;$$

$$C(s) = \frac{1}{2} k_2 s^2;$$

$$C(A) = \frac{1}{2} k_3 A^2.$$  

Where the coefficient $k_1, k_2, k_3$ respectively represents the carbon emission reduction coefficient, service effort coefficient and promotion effort coefficient. To simplify the calculation, we let $k_1 = k_2 = k_3 = 1$. $e, s, A$ respectively represents the level of emission reduction, service efforts and promotion efforts.

**Assumption 4.** The market demand function of the new retailer is: $D_n = \alpha e + \delta + \mu s$. Referencing to Chiang et al (2003), this paper lets the coefficient $\delta \in (0,1)$ to be convenient for computation and analysis, which denotes the consumers’ propensity to buy from the new retail channel. $\delta \rightarrow 0$ means the new retail channel is totally unacceptable, and $\delta \rightarrow 1$ represents there is no consumer choose the traditional retail channel. The market demand function of the new retailer is: $D_n = \alpha e + \delta + \mu s$. $\alpha$ represents the impact of carbon emission on demand of new retail channel, and $\mu$ stands for the impact of service efforts on the demand of new retail channel. The demand function of traditional retail channel is: $D_t = 1 - \delta + \beta e + \rho A$. $\beta$ represents the impact of carbon emission on demand of traditional retail channel, and $\rho$ stands for the impact of promotion efforts on demand of traditional retail channel.

### 4. Basic Model with Non-Cooperative Game

In this section, we establish two models of different supply chain structures: the decentralized scenario and centralized scenario, where the optimal reduction of carbon emissions, service and promotion are obtained.

#### 4.1 Decentralized Supply Chain Scenario

In the decentralized scenario, the manufacturer, the new retailer and the traditional retailer maximize their own profits in the decision-making process. At this time, the profit functions of the manufacturer, the new retailer and the traditional retailer are

$$\pi_m = -\frac{e^2}{2} + w \left(1 + e (\alpha + \beta) + s \mu + A \rho \right);$$

$$\pi_n = \frac{1}{2} k_1 e^2 + \delta + \mu s;$$

$$\pi_t = \frac{1}{2} k_3 A^2 + \beta e + \rho A.$$
we design different mode of cost sharing contracts, which consider the new retailer and traditional retailer to achieve optimal value in centralized situation. In order to improve the performance of the supply chain by environment than that of decentralized. As the members of the supply chain are independent rational, they want that in the decentralized scenario:

Theorem 1. The optimal decision maximizing profit of members is obtained \((e^{D^*}, s^{D^*}, A^{D^*})\). The optimal profits of members are \(\pi^{m}_{D^*}, \pi^{n}_{D^*}, \pi^{p}_{D^*}\), and the optimal supply chain profit is \(\pi_{sc}^{D^*}\).

Proof. The second derivative of \(e\) with respect to equation (4), pp. \(\partial^2 \pi_{w}/\partial e^2 = -1 < 0\), then the profit of the manufacturer is a strictly concave function with respect to the carbon reduction. Therefore, the only optimal carbon emission level can make the supplier profit maximize. Similarly, the profit of the new retailer and the traditional retailer is maximized by the only optimal service and the optimal promotion. The profit function takes the first derivative of its decision variable and equates it to zero. The optimal decision is:

\[
\pi^{m^*} = \frac{1}{2} w \left(2 + \alpha^2 + 2 \alpha \beta + \beta^2 + 2 \mu^2 I_s + 2 \rho^2 I_r\right);
\]
\[
\pi^{n^*} = \frac{1}{2} I_s \left(2 \left( (\alpha + \beta) + \mu \right) + \mu \right) I_s;
\]
\[
\pi^{p^*} = \frac{1}{2} I_r \left(2 \left( 2 \rho \beta - 2 \delta + \rho^2 I_r\right)\right);
\]
\[
\pi^{s^*} = \frac{1}{2} \left\{w \left(2 + w(\alpha + \beta)\right) + 2 \left(\delta + w(\alpha^2 + \alpha \beta + \mu^2)\right) I_s\right\} + \frac{1}{2} \left[\mu I_s^2 + 2 \left(1 - \delta + w(\alpha \beta + \beta^2 + \rho^2)\right) I_r + \rho^2 I_r^2\right] \}
\]

4.2 Centralized supply chain scenario

In the centralized scenario, the manufacturer and retailer maximize the entire system in the decision-making process. At this time, the profit function of entire supply chain is

\[
\pi_{sc} = \frac{e^2 + \alpha e I_s + \beta I_r}{2} + \left( (\alpha + \beta) + \mu \right) I_r \left(1 + e + \delta + A \right) I_r.
\]

Theorem 2. In the centralized scenario, the optimal decision maximizing profit of members is obtained \((e^{C^*}, s^{C^*}, A^{C^*})\) and the optimal supply chain profit is \(\pi_{sc}^{C^*}\).

Proof. The profit function of supply chain system is joint concavity with respect to above decision variable (More details in Appendix 1.1). \(\pi_{sc}^{C^*}\) takes the first derivative of the decision variable and equates it to zero, the optimal decision is:

\[
\pi^{m^*} = \frac{1}{2} w \left(2 + w(\alpha + \beta) + \beta I_r\right) = \mu I_s \left(1 + e + \delta + A \right) I_r.
\]

Substituting \(e^{C^*}, s^{C^*}, A^{C^*}\) to (7), we can obtain the optimal profits of members as follows.

\[
\pi^{m^*} = \frac{1}{2} w \left(2 + \alpha^2 + 2 \alpha \beta + \beta^2 + 2 \mu^2 I_s + 2 \rho^2 I_r\right);
\]
\[
\pi^{n^*} = \frac{1}{2} I_s \left(2 \left( (\alpha + \beta) + \mu \right) + \mu \right) I_s;
\]
\[
\pi^{p^*} = \frac{1}{2} I_r \left(2 \left( 2 \rho \beta - 2 \delta + \rho^2 I_r\right)\right);
\]
\[
\pi^{s^*} = \frac{1}{2} \left\{w \left(2 + w(\alpha + \beta)\right) + 2 \left(\delta + w(\alpha^2 + \alpha \beta + \mu^2)\right) I_s\right\} + \frac{1}{2} \left[\mu I_s^2 + 2 \left(1 - \delta + w(\alpha \beta + \beta^2 + \rho^2)\right) I_r + \rho^2 I_r^2\right] \}
\]

Proposition 1. The optimal decisions in the centralized scenario are in the following order in comparison to that in the decentralized scenario: \(e^{D^*} < e^{C^*}, s^{D^*} < s^{C^*}, A^{D^*} < A^{C^*}, \pi_{sc}^{D^*} < \pi_{sc}^{C^*}\).

Proof. By algebraic comparison, we can obtain as follows.

\[
e^{C^*} - e^{D^*} = \alpha I_s + \beta I_r > 0;
\]
\[
s^{C^*} - s^{D^*} = \mu I_s > 0;
\]
\[
A^{C^*} - A^{D^*} = \rho I_r > 0;
\]
\[
\pi_{sc}^{C^*} - \pi_{sc}^{D^*} = \frac{1}{2} \left(2 + w(\alpha^2 + \beta^2) + 2 \alpha \beta I_r + \beta^2 I_r^2\right) > 0.
\]

The results illustrate the centralized supply chain has a higher improvement on the economy and the environment than that of decentralized. As the members of the supply chain are independent rational, they want to achieve optimal value in centralized situation. In order to improve the performance of the supply chain by stimulating suppliers’ emission reduction levels, new retailer service levels and traditional retailer promotions, we design different mode of cost sharing contracts, which consider the new retailer and traditional retailer to
share the suppliers’ emission reduction costs (NTS), the supplier to share the new retailers’ service costs (NS), the supplier to share the traditional retailers’ promotion costs (ST) and costs sharing with each other (CS). The four modes are used to coordinate the low-carbon supply chain.

5. Coordinating Low-Carbon Supply Chain Contract

5.1 NTS Model

The supplier enhances reduction of carbon emissions to increase channel sales. We consider the new retailer and traditional retailer share the supplier’s costs of emission reduction. A cooperative emission reduction model is developed (NTS model). We denote the contract as \( (\varphi_1, \varphi_2) \), where \( \varphi_1 (0 < \varphi_1 < 1) \) is the cost fraction that the new retailer shares, \( \varphi_2 (0 < \varphi_2 < 1) \) is the cost fraction that the traditional retailer shares. In the NTS model, the profits of members are as follows.

\[
\pi_\alpha = -(1 - \varphi_1 - \varphi_2) \left(\frac{e^2}{2} + w(1 + e(\alpha + \beta) + s\mu + A\rho)\right);
\]

\[
\pi_\sigma = -\frac{e^2}{2} \varphi_1 \left(\frac{e^2}{2} + (\alpha \varphi + \delta + s\mu)L\right);
\]

\[
\pi_r = -\frac{\delta^2}{2} \varphi_2 \left(\frac{e^2}{2} + (1 + e\beta - \delta + A\rho)L\right).
\]

Theorem 3. If \( w < \lambda_n \), \( w < \lambda_r \), the optimal decision maximizing profit of members is obtained \( (e^{*\sigma}, s^{*\sigma}, A^{*\sigma}, \varphi_1^{*\sigma}, \varphi_2^{*\sigma}) \). The optimal profits of members are \( \pi_{m^{*\sigma}}, \pi_{n^{*\sigma}}, \pi_{r^{*\sigma}} \), and the optimal supply chain profit is \( \pi_{sc^{*\sigma}} \).

Proof. If \( w < \lambda_n, w < \lambda_r \), there exists optimal solution (More details in Appendix 2.1), and the obtain optimal decision variables are as follows.

\[
e^{*\sigma} = \alpha L_n + \beta L_r, \quad s^{*\sigma} = \mu L_n + \rho L_r,
\]

\[
\varphi_1^{*\sigma} = \frac{-w(\alpha + \beta) + 2\alpha L_n}{2(\alpha L_n + \beta L_r)}, \quad \varphi_2^{*\sigma} = \frac{-w(\alpha + \beta) + 2\beta L_r}{2(\alpha L_n + \beta L_r)}.
\]

Substituting the above values to (8), (9), (10), we can obtain the optimal profits of members and the supply chain as follows.

\[
\pi_{m^{*\sigma}} = \frac{1}{2} w \left(2 + (\alpha^2 + \alpha \beta + 2 \mu^2) L_n + (\alpha \beta + \beta^2 + 2 \rho^2) L_r\right);
\]

\[
\pi_{n^{*\sigma}} = \frac{1}{4} \left(2(\alpha^2 + \mu^2) L_n + w(\alpha + \beta) L_n + (\alpha \beta + \beta^2 + 2 \rho^2) L_r\right);
\]

\[
\pi_{r^{*\sigma}} = \frac{1}{4} \left(L_n \left(4w(\alpha + \beta) - 4\delta + 2(\beta^2 + \rho^2) L_r\right) + (\alpha \beta + \beta^2 + 2 \rho^2) L_r\right);
\]

\[
\pi_{sc^{*\sigma}} = \left(1 - \delta + w(\alpha \beta + \beta^2 + \rho^2) L_n + \frac{1}{2}(\beta^2 + \rho^2) L_r\right) + w(\alpha^2 + \alpha \beta + \mu^2) L_n + \frac{1}{2}(\alpha^2 + \mu^2) L_r.
\]

Proposition 2. If \( w < \lambda_n, w < \lambda_r, \) and \( 3\beta/\alpha > l_n/l_r > \beta/3\alpha \), in the case of cooperative emission reduction, the profits of supply chain members, the supply chain profits and suppliers’ emission reductions are higher than that in the decentralized scenario. The optimal service and promotion effort in the NTS model are also equal to that in the decentralized scenario with non-cooperative game. Namely, \( e^{*\sigma} > e^{D\sigma}, s^{*\sigma} = s^{D\sigma}, A^{*\sigma} = A^{D\sigma}, \pi_{m^{*\sigma}} > \pi_{m^{D\sigma}}, \pi_{n^{*\sigma}} > \pi_{n^{D\sigma}}, \pi_{r^{*\sigma}} > \pi_{r^{D\sigma}}, \pi_{sc^{*\sigma}} > \pi_{sc^{D\sigma}} \).

Proof. By algebraic comparison, we can obtain the following results.

\[
e^{*\sigma} - e^{D\sigma} = \alpha L_n + \beta L_r - w(\alpha + \beta) > 0
\]

\[
s^{*\sigma} = s^{D\sigma} = \mu L_n, \quad A^{*\sigma} = A^{D\sigma} = \rho L_r
\]

\[
\pi_{m^{*\sigma}} - \pi_{m^{D\sigma}} = \frac{1}{2} w(\alpha + \beta)(\alpha L_n + \beta L_r - w(\alpha + \beta)) > 0
\]

\[
\pi_{n^{*\sigma}} > \pi_{n^{D\sigma}}, \quad \pi_{r^{*\sigma}} > \pi_{r^{D\sigma}} \quad \text{(More details in Appendix A)}
\]

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\[ \pi_{sc}^{e^*} - \pi_{sc}^{D^*} = \frac{1}{2}(-w^2(\alpha + \beta)^2 + \alpha^2 I_n^2 + 2\alpha\beta l_n I_r + \beta^2 I_r^2) > 0 \]

From the Proposition 2, under the condition of \( w < l_n \), \( w < l_r \) and \( 3\beta/\alpha > l_n/l_r > \beta/3\alpha \), the new retailer and traditional retailer choose a cooperative emission reduction model. However, new retailer and traditional retailer will not improve their service and promotion effort, but the supplier will enhance the reduction of carbon emission to increase channel sales, so that each member of supply chain gets Pareto improvement to effectively improve the performance of supply chain.

5.2 NS Model

In order to the new retailer’s enhance service level to increase channel sales, the supplier shares the new retailer’s service costs. The supplier takes the lead in determining the share of the service cost, and the new retailer determines the service effort while the traditional retailer determines the promotion effort. A cooperative service model is developed (NS model). We denote the contract as \( (e^*, \epsilon^*) \). The optimal profits of members are respectively \( \pi_m^{e^*} \), \( \pi_n^{e^*} \), \( \pi_r^{e^*} \), and the optimal supply chain profit is \( \pi_{sc}^{e^*} \).

Theorem 4. If \( w < l_n \), there exists optimal solution (More details in Appendix 3.1), and the optimal decision variables are as follows.

\[ e^* = w(\alpha + \beta) \]

\[ \epsilon^* = \frac{1}{2}\mu (2w + l_r) \]

\[ A^- = \rho l_r \]

\[ \epsilon^* = \frac{2w - l_n}{2w + l_r} \]

Substituting the above values to (11), (12), (13), we can obtain the optimal profits of members and the supply chain as follows.

\[ \pi_m^{e^*} = \frac{1}{2} \left[ 2 + w\left( \alpha^2 + 2\alpha\beta + \beta^2 + \mu^2 \right) \right] \]

\[ \pi_n^{e^*} = \frac{1}{4} I_s \left[ 4\delta + 2w\left( 2\alpha^2 + 2\alpha\beta + \mu^2 \right) + \mu^2 I_s \right] \]

\[ \pi_r^{e^*} = \frac{1}{2} I_s \left[ 2 + 2w(\alpha + \beta) - 2\delta + \rho^2 I_r \right] \]

\[ \pi_{sc}^{e^*} = \left( \delta + w\left( \alpha^2 + \alpha\beta + \mu^2 \right) \right) I_n + \frac{3}{8} \mu^2 I_r^2 + \left[ \frac{1}{2} \left( 1 - \delta + w\left( \alpha\beta + \beta^2 + \rho^2 \right) \right) I_n + \rho^2 I_r^2 \right] \]

Proposition 3. If \( w < l_n < 2w \), in the case of cooperative service, the profits of supply chain members, the supply chain profits and the new retailers’ service effort are higher than that in the decentralized scenario. The traditional retailer’s optimal profit, service effort and promotion effort in the NS model are all equal to that in the decentralized scenario with non-cooperative game. Namely, \( e^{e^*} > e^{D^*} \), \( s^{e^*} = s^{D^*} \), \( A^{e^*} = A^{D^*} \), \( \pi_m^{e^*} > \pi_m^{D^*} \), \( \pi_n^{e^*} > \pi_n^{D^*} \), \( \pi_r^{e^*} > \pi_r^{D^*} \), \( \pi_{sc}^{e^*} > \pi_{sc}^{D^*} \).

Proof. By algebraic comparison, we can obtain the following results.

\[ e^{e^*} - e^{D^*} = 0 \]

\[ s^{e^*} - s^{D^*} = \frac{1}{2} \mu l_n > 0 \]
\[ A^e^* - A^D^* = 0; \]
\[ \pi_m^{s^*} - \pi_m^{D^*} = \frac{1}{8} \mu^2 (I_n - 2w)^2 > 0 \]
\[ \pi_n^{s^*} - \pi_n^{D^*} = \frac{1}{4} \mu^2 (2w - I_n)I_n > 0 \]
\[ \pi_r^{s^*} - \pi_r^{D^*} = 0 \]
\[ \pi_{sc}^{s^*} - \pi_{sc}^{D^*} = \frac{1}{8} \mu^2 (4w^2 - I_n^2) > 0 \]

From the Proposition 3, under the condition of \( w < I_n < 2w \), the supplier chooses the service cooperation model, but the supplier will not raise the carbon emission reduction level under this model. Since the traditional retailer is solely responsible for the cost of promotion, service cooperation between new retailer and supplier does not affect the promotion efforts of traditional retailer. After the supplier shares the cost of new retailer’s service efforts, new retailer will increase service levels and expand market demand, resulting in higher profits for supplier, new retailer and supply chain.

5.3 ST Model

In order to the traditional retailer’s enhance promotion level to increase channel sales, the supplier shares the traditional retailer’s promotion costs. The supplier takes the lead in determining the share of the promotion cost, and the tradition retailer determines the promotion effort while the new retailer determines the service effort. A cooperative promotion model is developed (ST model). We denote the contract as \((\theta)\), where \(\theta(0 < \theta < 1)\) is the cost fraction that the supplier shares. In the ST model, the profits of members are as follows.

\[ \pi_m = \frac{e^2}{2} - \theta \frac{A^2}{2} + w(1 + e(\alpha + \beta) + s\mu + A\rho) \]  
(14)

\[ \pi_n = -\frac{s^2}{2} + (ea + \delta + s\mu)I_n \]  
(15)

\[ \pi_r = -(1-\theta)\frac{A^2}{2} + (1 + e\beta - \delta + A\rho)I_r \]  
(16)

Theorem 5. If \( w < I_n \), the optimal decision maximizing profit of members is \((e^{P*}, s^{P*}, A^{P*}, \theta^*)\). The optimal profits of members are respectively \(\pi_{m}^{P*}\), \(\pi_{n}^{P*}\), \(\pi_{r}^{P*}\), and the optimal supply chain profit is \(\pi_{sc}^{P*}\).

Proof. If \( w < I_n \), there exists optimal solution (More details in Appendix 4.1), and the optimal decision variables are as follows.

\[ e^* = w(\alpha + \beta) \quad s^* = \mu I_n \quad A^* = \frac{1}{2} \rho (2w + I_n) \quad \theta^* = \frac{2w - I_n}{2w + I_n} \]

Substituting the above values to (14), (15), (16), we can obtain the optimal profits of members and the supply chain as follows.

\[ \pi_m^{s^*} = \frac{1}{8} 4w(2 + w(\alpha^2 + 2\alpha\beta + \beta^2 + \rho^2)) \]
\[ + 8w\mu^2 I_n + 4w\rho^2 I_n + \rho^2 I_n^2 \]

\[ \pi_n^{s^*} = \frac{1}{2} I_n (2w(\alpha + \beta) + \delta + \mu^2 I_n) \]

\[ \pi_r^{s^*} = \frac{1}{4} I_r (4 - 4\delta + 2w(2\alpha\beta + 2\beta^2 + \rho^2) + \rho^2 I_r) \]

\[ \pi_{sc}^{s^*} = \frac{1}{8} 4w(2 + w(\alpha^2 + 2\alpha\beta + \beta^2 + \rho^2)) \]
\[ + 8(\delta + w(\alpha^2 + \alpha\beta + \mu^2))I_n + 4\mu^2 I_n^2 + \]
\[ + 8(1 - \delta + w(\alpha\beta + \beta^2 + \rho^2))I_r + 3\rho^2 I_r^2 \]
Proposition 4. If \( w < l_r < 2w \), in the case of cooperative promotion, the profits of supply chain members, the supply chain profits and the traditional retailers’ service effort are higher than that in the decentralized scenario. The new retailer’s optimal profit, service effort and promotion effort in the NS model are all equal to that in the decentralized scenario with non-cooperative game. Namely, \( e^{\pi^*} = e^{D^*}; \ s^{\pi^*} = s^{D^*}; \ A_{\pi^*} > A_{D^*}; \ \pi_{m^*} > \pi_{m^{D^*}}; \ \pi_{p^*} > \pi_{p^{D^*}}; \ \pi_{sc^{D^*}} > \pi_{sc^{D^*}} \).

Proof. By algebraic comparison, we can obtain the following results.

\[
\begin{align*}
& e^{\pi^*} - e^{D^*} = 0 \\
& s^{\pi^*} - s^{D^*} = 0 \\
& A_{\pi^*} - A_{D^*} = pw - 0.5pl_r > 0 \\
& \pi_{n^*} - \pi_{n^{D^*}} = 0 \\
& \pi_{m^*} - \pi_{m^{D^*}} = \frac{1}{\theta^2} (\theta^2 - 2w + l_r)^2 > 0 \\
& \pi_{p^*} - \pi_{p^{D^*}} = \frac{1}{\theta^2} (2w - l_r)l_r > 0 \\
& \pi_{sc^*} - \pi_{sc^{D^*}} = \frac{1}{\theta^2} (4w^2 - l_r^2) > 0 \\
\end{align*}
\]

From the Proposition 4, under the condition of \( w < l_r < 2w \), the supplier chooses the promotion cooperation model, but the supplier will not raise the carbon emission reduction level under this model. Since the new retailer is solely responsible for the cost of service, promotion cooperation between traditional retailer and supplier does not affect the service efforts of new retailer. After the supplier shares the cost of traditional retailer’s promotion efforts, traditional retailer will increase promotion level and expand market demand, resulting in higher profits for supplier, traditional retailer and supply chain.

5.4 CS Model

Each member of the supply chain shares the effort costs, and traditional retailer considers sharing the investment cost of the supplier’s emission reduction. We denote the contract of new retailer as \( \phi_1(0 < \phi_1 < 1) \) and the contract of traditional retailer as \( \phi_2(0 < \phi_2 < 1) \). The supplier shares the cost of traditional retailer’s promotion effort, whose contract is \( 0(0 < \theta < 1) \). The supplier shares the cost of new retailer’s service effort, whose contract is \( \epsilon(0 < \epsilon < 1) \). In the CS model, the profits of members are as follows.

\[
\begin{align*}
\pi_n &= -\frac{\epsilon}{2} + w(1 + \epsilon (\alpha + \beta) + s\mu + A\rho) - (1 - \phi_1 - \phi_2) \frac{\epsilon^2}{2} - \theta \frac{A^2}{2} \quad (17) \\
\pi_s &= -(1 - \epsilon) - \phi_1 \frac{\epsilon^2}{2} - \phi_2 \frac{\epsilon^2}{2} + (\epsilon\alpha + \delta + s\mu)l_r \quad (18) \\
\pi_r &= -(1 - \theta) \frac{A^2}{2} - \phi_1 \frac{\epsilon^2}{2} - (1 - \epsilon\beta - \delta + A\rho)l_r \quad (19)
\end{align*}
\]

Theorem 6. If \( w < l_r < 2w \), \( w < l_a < 2w \), the optimal decision maximizing profit of members is \( (e^{\pi^*}, \ s^{\pi^*}, \ A^{\pi^*}, \ \phi_1^{***}, \ \phi_2^{***}, \ \theta^{***}, \ \epsilon^{***}) \). The optimal profits of members are respectively \( \pi_{m^{\pi^*}}; \pi_{n^{\pi^*}}; \pi_{r^{\pi^*}} \), and the optimal supply chain profit is \( \pi_{sc^{\pi^*}} \).

Proof. There exists optimal solution (More details in Appendix 5.1), and the optimal decision variables are as follows.

\[
\begin{align*}
\phi_1 &= \frac{2w - l_r}{2w + l_r} \quad \phi_2 = \frac{2w - l_a}{2w + l_a} \\
\phi_1^{***} &= \frac{-w (\alpha + \beta) + 2\alpha l_r}{2(\alpha l_r + \beta l_r)} \quad \phi_2^{***} = \frac{-w (\alpha + \beta) + 2\beta l_a}{2(\alpha l_a + \beta l_a)} \\
\epsilon^{***} &= \alpha l_r + \beta l_r \quad s^{***} = \frac{1}{2} \mu (2w + l_r) \quad A^{***} = \frac{1}{2} \rho (2w + l_r)
\end{align*}
\]

Substituting the above values to (17), (18), (19), we can obtain the optimal profits of members and the supply chain as follows.
If the marginal revenue of the supplier is greater than the threshold, the cooperation service mode and vice cooperation and promotion cooperation. Finally, compare and analyze the supply chain profits under the above four coordination models. Firstly, compare the supply chain profit under the CS mode with the supply chain profits under the other three models. Secondly, compare the supply chain profits under the service cooperation and promotion cooperation. Finally, compare the supply chain profits in the case of service cooperation, promotion cooperation and emission reduction cooperation, and draw conclusions on the proposition. (More details in Appendix 5.3)

From the Proposition 6, the supply chain has the largest profit under the CS mode. Each member of the supply chain bears the costs of each other’s efforts, thereby increasing the emission reduction level, the service effort level, promotion effort level, so that increasing the demand and the profits of all members and supply chain. If the marginal revenue of the supplier is greater than the threshold, the cooperation service mode and cooperative promotion mode are both superior to the cooperative emission reduction mode. On the contrary, the
cooperative emission reduction mode is second only to the CS mode. The relationship between $\mu$ and $\rho$ determines the supply chain profits under the cooperative service mode and cooperative promotion mode. Obviously, when the impact of service level on the channel demand of new retailer is greater than the impact of promotion level on the channel demand of traditional retailer, the coordination effect of the supplier’s choice to share the service cost of new retailer is better than that of choosing to share the promotion cost of traditional retailer, and vice versa.

6. Numerical Examples

6.1 Pareto Improvement of Supply Chain in Different Modes

In order to further analyze and compare the effectiveness of the four coordination mechanisms, and more intuitively reflect changes in the profit of the supplier, new retailer, traditional retailer and supply chain under different circumstances. The basic parameters are as follows: $\alpha = 1$, $\beta = 0.8$, $\mu = 0.5$, $\rho = 0.7$, $w = 1.7$, $I_r = 2.7$, $I_n = 3.3$, which guarantee equalization solutions for different modes.

Under the NTS mode, according to Theorem 1, the optimal supply chain strategy under decentralized decision-making is $(e^D, s^D, A^D) = (3.06, 1.65, 1.89)$. According to Theorem 3, the optimal supply chain strategy under cooperative emission reduction mode is $(e^*, s^*, A^*, \phi_1^*, \phi_2^*) = (5.46, 1.65, 1.89, 0.32, 0.12)$. Obviously, the decision variables in this mode and the decision variables under decentralized decision have the relationship as described in Proposition 2, which are as follows: $e^* > e^D$, $s^* = s^D$, $A^* = A^D$. The optimal profits of supplier, new retailer, traditional retailer and supply chain vary with consumers’ $\delta$ for new retail channel as shown in Figure 2. (For the sake of space, Figures 2-10 are all omitted.)

Figure 2 and Figure 3 demonstrate that regardless of the coefficient, the optimal profit of supplier, new retailer, traditional retailer and supply chain under cooperative emission reduction mode is greater than that of supplier, traditional retailer and supply chain under decentralized mode. Traditional retailer and new retailer share the cost of emission reduction from the supplier, which results in higher levels of supplier emission reduction and improves the profits of members and supply chain.

Under the NS mode, the optimal supply chain strategy under decentralized decision-making is $(e^D, s^D, A^D) = (3.06, 1.65, 1.89)$. According to Theorem 4, the optimal supply chain strategy under cooperative service mode is $(e^*, s^*, A^*, \epsilon^*) = (3.06, 1.68, 1.89, 0.02)$. Obviously, the decision variables in this mode and the decision variables under decentralized decision have the relationship as described in Proposition 3, which are as follows: $e^* = e^D$, $s^* > s^D$, $A^* = A^D$. The optimal profits of supplier, new retailer, traditional retailer and supply chain vary with consumers’ $\delta$ for new retail channel as shown in Figure 4 and Figure 5.

Figure 4 and Figure 5 demonstrate that regardless of the coefficient, the optimal profit of supplier, new retailer and supply chain under cooperative service mode is greater than that of supplier, traditional retailer and supply chain under decentralized mode. New retailer and supplier cooperate to enhance the service effort of new retailer to increase the channel demand of new retailing, which makes the profits of supplier and new retailer higher than that of supplier and new retailer under decentralized decision-making. Traditional retailer bears the cost of promotion effort alone, whose promotion effort level has not improved compared with decentralized decision-making situation under the service cooperation mode, and the channel demand for retailing has not changed. Therefore, the profit of traditional retailer remains unchanged, and $\pi_1^*$, $\pi_2^*$ completely coincide.

Under the ST mode, the optimal supply chain strategy under decentralized decision-making is $(e^D, s^D, A^D) = (3.06, 1.65, 1.89)$. According to Theorem 5, the optimal supply chain strategy under cooperative promotion mode is $(e^D, s^D, A^D, \theta^*) = (3.06, 1.65, 2.14, 0.12)$. The optimal profits of supplier, new retailer, traditional retailer and supply chain vary with consumers’ $\delta$ for new retail channel as shown in Figure 6 and Figure 7.

Figure 6 and Figure 7 demonstrate that regardless of the coefficient, the optimal profit of supplier, traditional retailer and supply chain under cooperative promotion mode is greater than that of supplier, traditional retailer and supply chain under decentralized mode. Promotion cooperation enables traditional retailer to increase the channel demand of traditional retailer, thereby increasing the channel demand for traditional retailing, which makes the profits of supplier and traditional retailer higher than that of supplier and traditional retailer under decentralized decision-making. New retailer bears the cost of service effort alone, whose service effort level has not improved compared with decentralized decision-making situation under the promotion cooperation mode, and the channel demand for new retailing has not changed. Therefore, the profit of new retailer remains unchanged, and $\pi_1^*$, $\pi_2^*$ completely coincide.

Under the CS mode, the optimal supply chain strategy under decentralized decision-making is $(e^D, s^D, A^D) = (3.06, 1.65, 1.89)$. According to Theorem 6, the optimal supply chain strategy under cost
sharing mode is \((e^*, s^*, A^*, \varphi_1^*, \varphi_2^*, \theta^*, \varepsilon^*) = (5.46, 1.68, 2.14, 0.32, 0.12, 0.12, 0.02)\). Obviously, the decision variables in this mode and the decision variables under decentralized decision have the relationship as described in Proposition 5, which are as follows: \(e^* > e^D\), \(s^* > s^D\), \(A^* > A^D\). The optimal profits of supplier, new retailer, traditional retailer and supply chain vary with consumers’ \(\delta\) for new retail channel as shown in Figure 8 and Figure 9.

Figure 8 and Figure 9 demonstrate that regardless of the coefficient, the optimal profit of supplier, traditional retailer and supply chain under cost sharing mode is greater than that of supplier, traditional retailer and supply chain under decentralized mode. Traditional retailer and new retailer share the supplier’s emission reduction costs, and the supplier shares new retailer’s service costs and traditional retailer’s promotion costs, which lead to increase the supplier’s emission reduction level, service efforts level and promotion efforts level. The increase in efforts has led to an increase in demand, thus improving the profits of supply chain and members.

Figure 2, Figure 4, Figure 6 and Figure 8 demonstrate the profit of new retailer is positively correlated with new channel preference coefficient of consumer, and the profit of traditional retailer decreases as the consumer’s new channel preference coefficient increases. Since traditional retailer has obtained a certain market through promotion efforts, traditional retailer’s profit is not equal to 0 when the consumer’s new channel preference coefficient is 1. Figure 3, Figure 5, Figure 7 and Figure 9 demonstrate supply chain profits increase with the increase in consumer’s new channel preference, and opening up a ‘new retailing’ channel can boost supply chain profit.

6.2 Comparison of Supply Chain Profit Under Different

When \(w\) and \(l_1, l_2, \mu\) and \(\rho\) satisfy different relations, there are differences in the order of supply chain profit under the four modes. Considering the proposition 6 has given detailed proof, this study discusses one of the relations, and the parameters set are the same as above.

Figure 10 reflects the supply chain profit of the four coordination modes. The supply chain profit in the four modes from the largest to the smallest: the cost sharing mode, the cooperative emission reduction mode, the cooperative promotion mode and the cooperative service mode. The supply chain profit under the four cooperation modes is greater than that under the decentralized decision, but the coordination effect of cooperative service mode and cooperative promotion mode are both general, both of which has little effect on the supply chain profit. In addition, the coordination effect of cooperative emission reduction mode is close to the mode of cost sharing mode. Therefore, when the supplier’s profitability decreases, traditional retailer and new retailer should actively share the emission reduction cost of supplier to stimulate the supplier’s emission reduction level, and the supply chain profit will be improved significantly.

6.3 Change of share ratio in the cost sharing mode

According to the model in this paper, the trend of coordination ratio and related parameters in cost sharing mode is shown in Figure 11.

- **Figure 11.** Trend in the proportion of emission reduction and related factors under the cost sharing mode
In Figure 11, α represents the impact of emission reduction on the channel demand for new retailer, β represents the impact of emission reduction on the channel demand for retailer. The cost sharing ratio of new retailer’s emission reduction is positively correlated with α, but the cost sharing ratio of traditional retailer’s emission reduction is negatively correlated with α. The cost sharing ratio of traditional retailer’s emission reduction is positively correlated with β, but the cost sharing ratio of new retailer’s emission reduction is negatively correlated with β. If the impact factor of emission reduction on new retailing’s channel demand is greater than that of emission reduction on retailing’s channel demand, the cost sharing ratio of new retailer’s emission reduction is greater than that of traditional retailer’s emission reduction, and vice versa. Therefore, the impact factor of emission reduction on channel demand directly affects the cost sharing ratio of channel retailer’s emission reduction, and channel retailer with greater impact factor shares high cost of the supplier’s emission reduction, which stimulates the supplier’s emission reduction level, expands channel demand and improves supply profit.

7. Conclusions

This paper explores the optimal decision-making and profitability of supply chain members under centralized and decentralized decision-making by constructing a dual-channel supply chain model consisting of supplier, new retailer and traditional retailer. On this basis, four supply chain contract mechanisms are designed from the perspective of cost sharing, and the degree of Pareto improvement is discussed. Through model comparison and numerical examples, the conclusions are as follows: 1) Whether it is service cooperation, promotion cooperation or emission reduction cooperation, the one-way cost sharing coordination mechanism is not enough for each member’s incentives, which should bear the other’s effort cost to effectively improve the Pareto improvement degree. 2) Supply chain profit is positively related to consumer’s new channel preference. In the ‘Internet +’ era, ‘new retail’ breaks the traditional retailing model from the mode, service and experience. It traditional retailer wants to retain their market share and gain more revenue, they should actively open up a ‘new retail’ model to increase their profit while increasing supply profit. 3) When the marginal benefit of the supplier is greater than the threshold, the cooperative emission reduction mode has the worst coordination effect. On the contrary, the coordination effect is second only to cost sharing mode. 4) If the impact of service level on the demand of new retail channel is higher than that of promotion level on the demand of traditional retail channel, the cooperative service mode coordination effect is better than the cooperative promotion mode, and vice versa. 5) Under the cooperation mode of cost sharing, the proportion of emission reduction cost of channel retailer is positively related to the impact of emission reduction on the channel, and the channel retailer that emission reduction has a greater impact on channel demand, will share the higher emission reduction cost of supplier, which promotes emission reduction, stimulates demand and increases profit.

This paper only considers the situation of deterministic demand when constructing the model, so we can consider the situation of stochastic demand in the future. In addition, the government diversifies its means of controlling carbon emission, so the decision-making of supply chain members is influenced by the government’s carbon emission policy. In the future, it can be further studied in conjunction with the government’s carbon emission policy.

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Appendix A

1.1 π_{sc} is a joint concave function for e, s, A.

Proof. According to formula (7), the Hessian matrix π_{sc} is got.

\[
H(e, s, A) = \begin{bmatrix}
\frac{\partial^2 \pi_{sc}}{\partial e \partial s} & \frac{\partial^2 \pi_{sc}}{\partial e \partial A} & \frac{\partial^2 \pi_{sc}}{\partial s \partial A} \\
\frac{\partial^2 \pi_{sc}}{\partial e \partial s} & \frac{\partial^2 \pi_{sc}}{\partial e \partial A} & \frac{\partial^2 \pi_{sc}}{\partial s \partial A} \\
\frac{\partial^2 \pi_{sc}}{\partial e \partial s} & \frac{\partial^2 \pi_{sc}}{\partial e \partial A} & \frac{\partial^2 \pi_{sc}}{\partial s \partial A}
\end{bmatrix}
\]

Because of \(\frac{\partial^2 \pi_{sc}}{\partial e^2} < 0\), there is a second order determinant greater than 0 and |H| > 0, and π_{sc} is a joint concave function for e, s, A, so the supply chain profit has an optimal solution.

2.1 The optimal solution under cooperative emission reduction mode

Proof. New retailer and traditional retailer determine the cost sharing ratio, and the supplier determines the emission reduction effort, so the inverse induction method is used to solve it. 1 – ϕ₁ – ϕ₂ was the proportion of
emission reduction ratio assumed for the supplier, we found 1 - \varphi_1 - \varphi_2 > 0. According to formula (8), solving the second-order partial derivative of \(e\), we found \(\frac{\partial^2 \pi_n}{\partial e^2} = -(1 - \varphi_1 - \varphi_2) < 0\). Solving the first-order partial derivative of \(e\) and let it to be 0, \(e = \frac{w(a + b)}{1 + \varphi_1 + \varphi_2}\) was got. Putting the above results into formulas (9) and (10), formulas (19) and (20) were obtained.

\[
\begin{align*}
\pi_n &= -\frac{s^2}{2} - \frac{w^2(\alpha + \beta)^2 \varphi_1}{2(1 + \varphi_1 + \varphi_2)} + \ln(\delta + s\mu - \frac{wa(\alpha + \beta)}{1 + \varphi_1 + \varphi_2}) \\ 
\pi_r &= -\frac{s^2}{2} - \frac{w^2(\alpha + \beta)^2 \varphi_2}{2(-1 + \varphi_1 + \varphi_2)^2} + \ln(1 - \delta + Ap - \frac{w(\alpha + \beta)}{1 + \varphi_1 + \varphi_2}) \\
\end{align*}
\]

(19)  (20)

The Hessian matrix for \(\pi_n\) was as follows:

\[
H(s, \varphi_1) = \begin{bmatrix}
\frac{\partial^2 \pi_n}{\partial s^2} & \frac{\partial^2 \pi_n}{\partial s \partial \varphi_1} \\
\frac{\partial^2 \pi_n}{\partial s \partial \varphi_2} & \frac{\partial^2 \pi_n}{\partial \varphi_1^2}
\end{bmatrix}
\]

To ensure \(H(s, \varphi_1)\) was negative, 
\[
|H(s, \varphi_1)| = \frac{w(a + b)(w(a + b)(2 + \varphi_1 - 2\varphi_2) + 2a_1\varphi_1(-1 + \varphi_1 + \varphi_2))}{(1 + \varphi_1 + \varphi_2)^4}
\]

was got. Supposing \(f_1(\varphi_1, \varphi_2) = 2 + \varphi_1 - 2\varphi_2\), we found \(f_1(\varphi_1, \varphi_2)\) monotonically increased in the region of \(0 < \varphi_1, \varphi_2 < 1\) and \(f_1(\varphi_1, \varphi_2) > f(0,0) = 2\). Therefore, if \(0 < \frac{a_1\varphi_1}{w(a + b)} < 1 \Leftrightarrow w < l_n\), \(\pi_n\) was a joint concave function for \(s, \varphi_1\), and the optimal solution for the profit of new retailer was got.

The Hessian matrix for \(\pi_r\) was:

\[
H(A, \varphi_2) = \begin{bmatrix}
\frac{\partial^2 \pi_r}{\partial A^2} & \frac{\partial^2 \pi_r}{\partial A \partial \varphi_2} \\
\frac{\partial^2 \pi_r}{\partial \varphi_2 \partial A} & \frac{\partial^2 \pi_r}{\partial \varphi_2^2}
\end{bmatrix}
\]

To ensure \(H(A, \varphi_2)\) was negative, 
\[
|H(A, \varphi_2)| = \frac{w(a + b)(w(a + b)(2 + 2\varphi_1 - 2\varphi_2) + 2a_1\varphi_2(-1 + \varphi_1 + \varphi_2))}{(1 + \varphi_1 + \varphi_2)^4}
\]

was got. Supposing \(f_2(\varphi_1, \varphi_2) = 2 + 2\varphi_1 - 2\varphi_2\), we found \(f_2(\varphi_1, \varphi_2)\) monotonically increased in the region of \(0 < \varphi_1, \varphi_2 < 1\) and \(f_2(\varphi_1, \varphi_2) > f(0,0) = 2\). Therefore, if \(0 < \frac{\beta_1\varphi_2}{w(a + b)} < 1 \Leftrightarrow w < l_1\), \(\pi_r\) was a joint concave function for \(A, \varphi_2\), and the optimal solution for the profit of traditional retailer was got.

2.2 if \(w < l_n\), \(w < l_1\) and \(3\beta/\alpha > l_n/l_1 > \beta/3\alpha\), then \(\pi_n^{e*} > \pi_n^{D*}\) and \(\pi_r^{e*} > \pi_r^{D*}\) were got.

Proof: \(\pi_n^{e*} - \pi_n^{D*} = \frac{1}{4}(2\beta_1^2l_1^2 + 2a_1l_1\beta_1 + bl_1w(a + b) - 3a_1w(a + b)) > \frac{1}{4}(3a_1l_1\beta_1 - \alpha_1^2l_2^2)\)

Assuming that the marginal revenue is \(3\beta_1 > \alpha_1l_1\), we found \(\frac{1}{4}(3a_1l_1\beta_1 - \alpha_1^2l_2^2) > 0\), namely \(\pi_n^{e*} > \pi_n^{D*}\).

\(\pi_r^{e*} - \pi_r^{D*} = \frac{1}{4}(2\beta_1^2l_1^2 + 2a_1l_1\beta_1 + bl_1w(a + b) - 3a_1w(a + b)) > \frac{1}{4}(3a_1l_1\beta_1 - \beta_1^2l_2^2)\)

Assuming that the marginal revenue is \(3a_1l_1 > \beta_1l_1\), we found \(\frac{1}{4}(3a_1l_1\beta_1 - \beta_1^2l_2^2) > 0\), namely \(\pi_r^{e*} > \pi_r^{D*}\).

3.1 if \(w < l_n\), in the cooperative service mode, the model has an optimal solution

Proof: According to the inverse induction method, we found the second-order partial derivative of s based on formula (12) is \(\frac{\partial^2 \pi_n}{\partial s^2} = -(1 - e) < 0\), and obtained the second-order partial derivative of A based on formula (13) is \(\frac{\partial^2 \pi_n}{\partial A^2} = -1 < 0\), so both new and traditional retailer have the optional profit. According to formulas (14) and (15), we found the first-order partial derivative of s and A, and then let them to be 0. Finally, we got \(s = -\frac{\mu}{1-e}\). Putting \(A = l_1\) into formula (11), then we obtained \(\pi_m = -\frac{e^2}{2} + w + ew(a + b) + Awp - \frac{w^2(\alpha + \beta)^2}{2(-1+e)^2}\)

Hessian matrix \(H(e, \varepsilon)\) about \(\pi_m\) was got.
\[
H(e, \varepsilon) = \begin{bmatrix}
\frac{\partial^2 \pi_m}{\partial e^2} & \frac{\partial^2 \pi_m}{\partial e \partial \varepsilon} \\
\frac{\partial^2 \pi_m}{\partial e \partial \varepsilon} & \frac{\partial^2 \pi_m}{\partial \varepsilon^2}
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & -\frac{\mu^2 l_i (2w(-1 + \varepsilon) + (2 + \varepsilon)l_r)}{(-1 + \varepsilon)^4}
\end{bmatrix}
\]

To ensure \(H(e, \varepsilon)\) was negative, \(|H(e, \varepsilon)| = \frac{\mu^2 l_i (2w(-1 + \varepsilon) + (2 + \varepsilon)l_r)}{(-1 + \varepsilon)^4} > 0\) should be satisfied, which mean \(2w(-1 + \varepsilon) + (2 + \varepsilon)l_r > 0\), and \(\frac{2w}{1 + \varepsilon}\) was also got. Supposing \(f_3(\varepsilon) = \frac{2w}{1 + \varepsilon}\), we found \(f_3(\varepsilon)\) monotonically increased in the region of \(0 < \varepsilon < 1\) and \(f_3(\varepsilon) > f_3(0) = 2\). Therefore, if \(w < l_r, \pi_m\) was a joint concave function for \(e\) and \(\varepsilon\), and the optimal solution for the profit of retailer was got.

4.1 if \(w < l_r\), in the cooperative service mode, the model has an optimal solution

Proof. According to the inverse induction method, we found the second-order partial derivative of \(A\) based on formula (16) is \(\frac{\partial^2 \pi_m}{\partial A^2} = -(1 - \theta) < 0\), and obtained the second-order partial derivative of \(s\) based on formula (15) is \(\frac{\partial^2 \pi_m}{\partial s^2} = -1 < 0\), so both new and traditional retailer have the optimal profit. According to formulas (15) and (16), we found the first-order partial derivative of \(s\) and \(A\), and then let them to be 0. Finally, we got \(s = \mu l_l\). Putting \(A = -\frac{\mu l_r}{1 + \theta}\) into formula (14), then we obtained

\[
\pi_m = -\frac{e^2}{2} + \frac{e^2 w^2}{2} + w(1 + e(\alpha + \beta) + \mu^2 l_l - \rho^2 l_r).
\]

Hessian matrix \(H(e, \theta)\) about \(\pi_m\) was got.

\[
H(e, \theta) = \begin{bmatrix}
\frac{\partial^2 \pi_m}{\partial e^2} & \frac{\partial^2 \pi_m}{\partial e \partial \theta} \\
\frac{\partial^2 \pi_m}{\partial e \partial \theta} & \frac{\partial^2 \pi_m}{\partial \theta^2}
\end{bmatrix} = \begin{bmatrix}
-1 & 0 \\
0 & -\rho^2 l_r (2w(-1 + \theta) + (2 + \theta)l_r)
\end{bmatrix}
\]

To ensure \(H(e, \theta)\) was negative, \(|H(e, \theta)| = \rho^2 l_r (2w(-1 + \theta) + (2 + \theta)l_r) > 0\) should be satisfied, which mean \(2w(-1 + \theta) + (2 + \theta)l_r > 0\), and \(\frac{2w}{1 + \theta}\) was also got. Supposing \(f_4(\theta) = \frac{2w}{1 + \theta}\), we found \(f_4(\theta)\) monotonically increased in the region of \(0 < \theta < 1\), and \(f_4(\theta) > f_4(0) = 2\). Therefore, if \(w < l_r, \pi_m\) was a joint concave function for \(e\) and \(\theta\), and the optimal solution for the profit of supplier was got.

5.1 if \(w < l_r, w < l_r < 2w\) in the cost sharing model, the model has an optimal solution

Proof. Let \(\partial \pi_m/\partial e = 0\) in formula (17), then we got \(e = -\frac{w(\alpha + \beta)}{1 + \theta}\). Let \(\partial \pi_m/\partial s = 0\) in formula (18), then we obtained \(s = -\frac{\mu l_l}{1 + \varepsilon}\). Let \(\partial \pi_m/\partial A = 0\) in formula (19), then we got \(A = -\frac{\mu l_r}{1 + \theta}\). Putting \(s = -\frac{\mu l_l}{1 + \varepsilon}\) and \(A = -\frac{\mu l_r}{1 + \theta}\) into \(\pi_m\), then we got the Hessian matrix for \(\pi_m\) was

\[
H(e, \varepsilon, \theta) = \begin{bmatrix}
-1 + \phi_1 + \phi_2 & 0 \\
0 & -\frac{\mu^2 l_i (2w(-1 + \varepsilon) + (2 + \varepsilon)l_r)}{(-1 + \varepsilon)^4}
\end{bmatrix}
\]

\[
H(e, \varepsilon, \theta)\) was negative if \(w < l_r\) and \(w < l_r\), then \(\pi_m\) was a joint concave function for \(e, \varepsilon\) and \(\theta\). Putting \(e = -\frac{w(\alpha + \beta)}{1 + \phi_1 + \phi_2}\) into \(\pi_m\), then we got Hessian matrix \(H(s, \phi_1)\) about \(\pi_n\).

\[
H(s, \phi_1) = \begin{bmatrix}
\frac{\partial^2 \pi_n}{\partial s^2} & \frac{\partial^2 \pi_n}{\partial s \partial \phi_1} \\
\frac{\partial^2 \pi_n}{\partial s \partial \phi_1} & \frac{\partial^2 \pi_n}{\partial \phi_1^2}
\end{bmatrix} = \begin{bmatrix}
-1 + \varepsilon & 0 \\
0 & -\frac{w(\alpha + \beta)(w(\alpha + \beta)(2\phi_1 - 2\phi_2) + 2\theta l_r(-1 + \phi_1 + \phi_2))}{(-1 + \phi_1 + \phi_2)^4}
\end{bmatrix}
\]

\(H(s, \phi_1)\) was negative if \(w < l_r\), then \(\pi_n\) was a joint concave function for \(s\) and \(\phi_1\).

Putting \(e = -\frac{w(\alpha + \beta)}{1 + \phi_1 + \phi_2}\) into \(\pi_n\), then we obtained Hessian matrix \(H(A, \phi_2)\) about \(\pi_r\).

\[
H(A, \phi_2) = \begin{bmatrix}
\frac{\partial^2 \pi_r}{\partial A^2} & \frac{\partial^2 \pi_r}{\partial A \partial \phi_2} \\
\frac{\partial^2 \pi_r}{\partial A \partial \phi_2} & \frac{\partial^2 \pi_r}{\partial \phi_2^2}
\end{bmatrix} = \begin{bmatrix}
-1 + \theta & 0 \\
0 & -\frac{w(\alpha + \beta)(w(\alpha + \beta)(-2 + 2\phi_1 - 2\phi_2) - 2\theta l_r(-1 + \phi_1 + \phi_2))}{(-1 + \phi_1 + \phi_2)^3}
\end{bmatrix}
\]

\(H(A, \phi_2)\) was negative if \(w < l_r\), then \(\pi_r\) was a joint concave function for \(A\) and \(\phi_2\).

5.2 \(\pi_m^r > \pi_m^{D_r}, \pi_m^r > \pi_m^{D^r}, \pi_r^r > \pi_r^{D_r}, \pi_r^r > \pi_r^{D^r}, \pi_{sc}^r > \pi_{sc}^{D_r}, \pi_{sc}^r - \pi_m^{D^r} = \mu^2 (2w - l_l)^2 + \rho^2 (2w - l_r)^2 + 4w(\alpha + \beta)(\alpha l_n + \beta l_r - w(\alpha + \beta)) > 0\)
was obtained. If \( w \geq \sqrt{\frac{\mu_1^2 + 4(\alpha_1 + \beta_1)^2}{4\rho^2 + 4(\alpha + \beta)^2}} \) and \( \mu < \rho \), then \( \pi_{sc}^{**} < \pi_{sc}^{*} < \pi_{sc}^{**} < \pi_{sc}^{**} \) was obtained. If \( w < \min \left\{ \sqrt{\frac{\mu_1^2 + 4(\alpha_1 + \beta_1)^2}{4\rho^2 + 4(\alpha + \beta)^2}}, \sqrt{\frac{\mu_1^2 + 4(\alpha_1 + \beta_1)^2}{4\rho^2 + 4(\alpha + \beta)^2}} \right\} \) and \( \mu < \rho \), then \( \pi_{sc}^{**} < \pi_{sc}^{**} < \pi_{sc}^{**} < \pi_{sc}^{**} \) was obtained. If \( w < \min \left\{ \sqrt{\frac{\mu_1^2 + 4(\alpha_1 + \beta_1)^2}{4\rho^2 + 4(\alpha + \beta)^2}}, \sqrt{\frac{\mu_1^2 + 4(\alpha_1 + \beta_1)^2}{4\rho^2 + 4(\alpha + \beta)^2}} \right\} \) and \( \mu > \rho \), then \( \pi_{sc}^{**} < \pi_{sc}^{**} < \pi_{sc}^{**} < \pi_{sc}^{**} \) was obtained.
References


